Continuum dislocation-based modelling of length-scale dependent plasticity on the microscale

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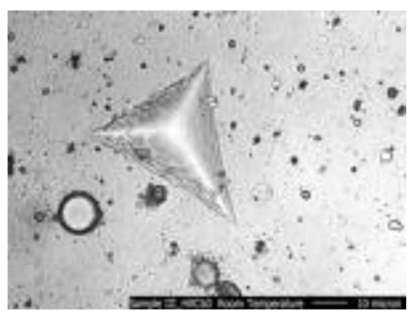
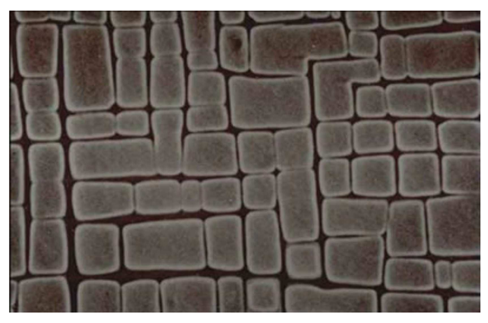
Problem setting

Plastic deformation on the scale of a few microns is length-scale dependent: **smaller is harder**

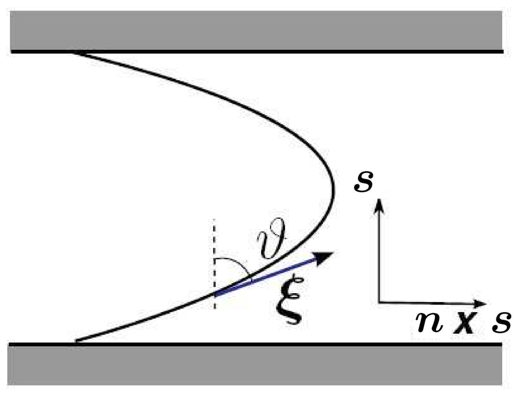
Typical application fields:
MEMS, nanoindentation, thin films,
materials with a microstructure of hard and soft phases
(e.g. steels, Ni-base superalloys)





Modelling approach:
The plastic deformation is traced back to the motion of dislocations. Not discrete dislocations are described, but evolution equations for a small number of field variables are derived that describe their collective motion, yielding a continuum model. The crucial point is to take into account the line tension of the dislocations, which becomes important on the microscale. Crystal plasticity enables a coupling of this model to small strain continuum mechanics.

A representative dislocation is described as **parametric curve,** thus providing its curvature and hence the **self-force** on the dislocation.



Equilibrium of forces → velocity field:

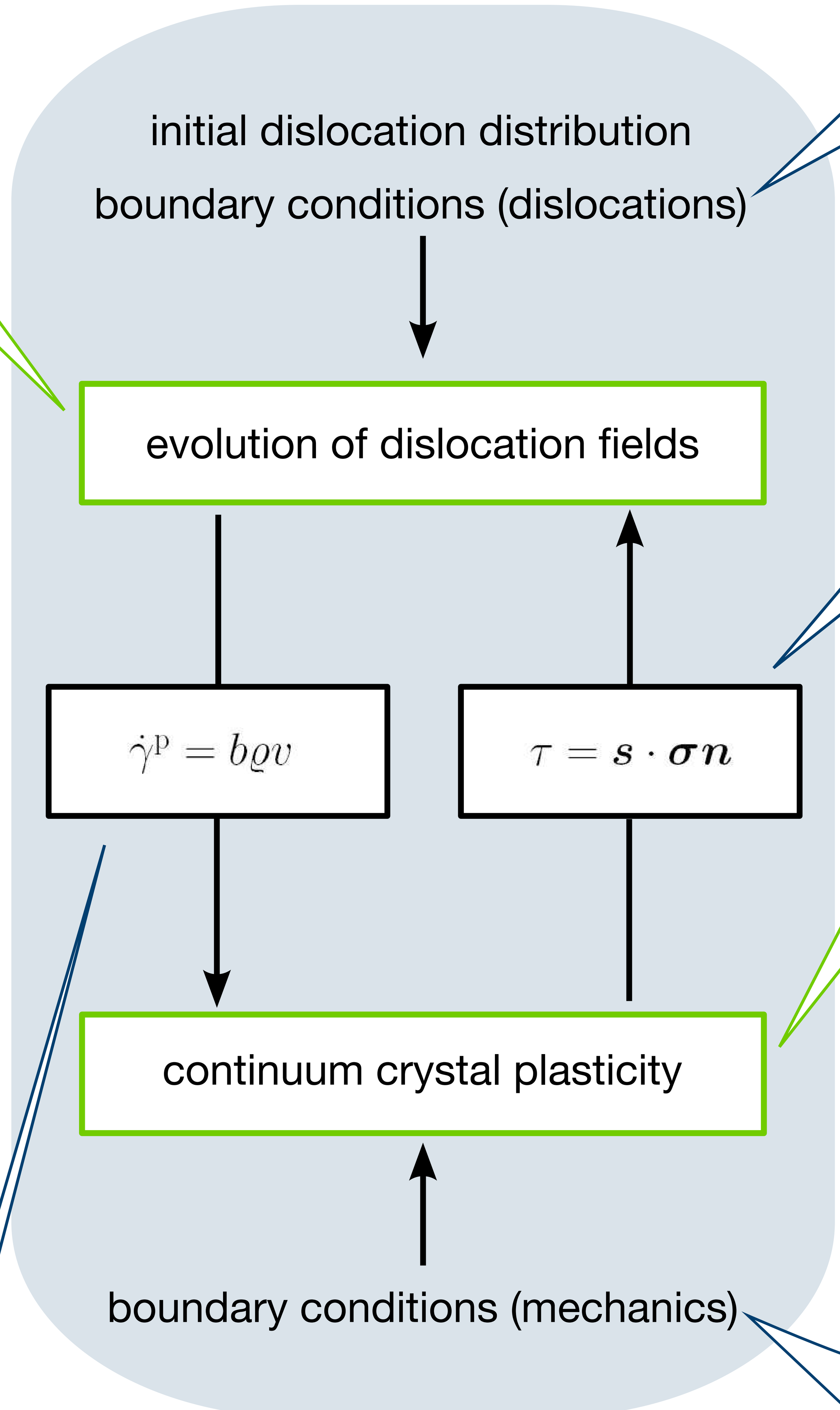
$$Bv = (\tau - \hat{\tau})b + T\kappa$$

Continuous distributions of dislocations in form of **single-valued fields** are considered (unique line orientation ϑ and density ϱ). Their superposition enables a description of **multiple-valued fields**.

The standard continuum theory of dislocations together with the parametric description yields **evolution equations** for the field variables:

$$\frac{\partial \varrho}{\partial t} = -\mathrm{div}(\varrho v \boldsymbol{\nu}) - \varrho v \kappa \ , \quad \frac{\partial \vartheta}{\partial t} = \nabla_{\boldsymbol{\xi}} v - v \, \mathrm{div}\boldsymbol{\xi}$$

The **plastic slip** γ^{p} is calculated by means of the Orowan equation based on the Burgers vector magnitude, the dislocation density and the displacement of a representative dislocation.



Boundary conditions for the field evolution equations account for the confinements of the dislocation motion due to the inhomogeneous loading or microstructure.

On impenetrable phase interfaces: $v = 0$
On free surfaces (image force): $\vartheta = 0$

Thus the bow-out of the dislocations is enforced.

Schmid's law is applied to calculate the resolved shear stress τ on the considered slip systems (normal $\boldsymbol{n}$, slip direction $\boldsymbol{s}$) from the macroscopic stress tensor.

The macroscopic tensor of the **plastic distortion rate** $\dot{\boldsymbol{\beta}}^{\mathrm{p}}$ is assembled from the plastic slip contributions of the active slip systems:

$$\dot{\boldsymbol{\beta}}^{\mathrm{p}} = \sum \dot{\gamma}^{\mathrm{p}} \boldsymbol{s} \otimes \boldsymbol{n}$$

Investigated **loading conditions**:
- shearing of thin films
- bending of free-standing crystalline strips
- torsion of thin wires
- shearing of a representative volume element of a composite

Numerical implementation

The numerical implementation of the model is based on (1) the finite element method applied to the mechanical problem, (2) a vertical line method applied to the Lagrangian formulation of the evolution equations, which form a convection dominated system (Lagrangian particle tracking method), and (3) a homogenisation step for their coupling.

References

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