Measurement of all six components of X-ray elastic factors

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Motivation

The best way to measure the full stress tensor in the material with the texture as shown in Figure 1 is:
1.) Measure lattice plane distances in directions 1 to 9, where intensity is highest.
2.) Calculate $\varepsilon\,(\boldsymbol{r},hkl) = (d(\boldsymbol{r},hkl)-d_0)/d_0$.
3.) Build the matrix equation

$$\begin{pmatrix}\varepsilon(\boldsymbol{r}_1,220)\\ \varepsilon(\boldsymbol{r}_2,220)\\ \cdot\\ \cdot\\ \cdot\\ \varepsilon(\boldsymbol{r}_N,220)\end{pmatrix}=\begin{pmatrix}\boldsymbol{F}_{11}(\boldsymbol{r}_1,220) & \boldsymbol{F}_{22}(\boldsymbol{r}_1,220) & \cdots & 2\boldsymbol{F}_{12}(\boldsymbol{r}_1,220)\\ \boldsymbol{F}_{11}(\boldsymbol{r}_2,220) & \boldsymbol{F}_{22}(\boldsymbol{r}_2,220) & \cdots & 2\boldsymbol{F}_{12}(\boldsymbol{r}_2,220)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \boldsymbol{F}_{11}(\boldsymbol{r}_N,200) & \boldsymbol{F}_{22}(\boldsymbol{r}_N,200) & \cdots & 2\boldsymbol{F}_{12}(\boldsymbol{r}_N,200)\end{pmatrix}\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{pmatrix} \qquad (1)$$

and solve it for σ_{ij}.
For that aim the $\boldsymbol{F}$-tensors must be known for the directions 1 to 9.

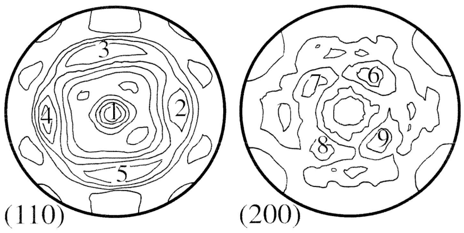

Fig. 1

Measurement of *F* (*r*, *hkl*)

The principle for the measurement of an ***F***-tensor is to establish a system of linear equations similar to Eq. (1) but with F_{ij} and σ_{ij} interchanged. This is possible because of the symmetry in ***F*** and σ of the basic equation

$$\varepsilon(\boldsymbol{r},hkl)=F_{ij}(\boldsymbol{r},hkl)\,\sigma_{ij}\,. \qquad (2)$$

With at least six – properly chosen – stress states and ε measured each time in the direction ***r*** the components of ***F*** can be calculated.

Since the texture of Figure 1 obviously has a fourfold symmetry, it is sufficient to measure the ***F***-tensors for the directions 1, 2 and 6. The ***F***-tensors for the directions 3, 4, 5 can be calculated from those of direction 2 by a tensor transformation, whereas ***F*** for directions 7, 8, 9 can be obtained by the same tensor transformation from 6.

Generation of different stress states

The specimen was shaped as a cuboctahedron, Figure 2a.
Compressive forces applied on each of the seven pairs of parallel faces yield seven different stress states. The direction of these forces are symbolized by $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ etc. in Figure 2a.
For the direction $\boldsymbol{s}_1$ which is parallel to the *z*-axis, the stress tensor is given by

$$\{\sigma_{ij}(\boldsymbol{s}_1)\}=\begin{pmatrix}\sigma_{\mathrm{I}} & 0 & 0\\ 0 & \sigma_{\mathrm{I}} & 0\\ 0 & 0 & \sigma_{\mathrm{III}}\end{pmatrix}. \qquad (3)$$

This stress tensor was calculated using the finite element method and is present for the centre of the cuboctahedral specimen. An example of a calculated stress distribution is shown in Figure 3.

For a compressive force applied on any other pair of parallel faces the stress tensor is calculated by the tensor transformation

$$\sigma_{ij}(\boldsymbol{s},hkl)=D_{ik}(\boldsymbol{s})\sigma_{kl}(\boldsymbol{s}_1)D_{jl}(\boldsymbol{s})\,. \qquad (4)$$

With these seven different stress tensors a well conditioned system of linear equations for the six independent components of the ***F***-tensor is obtained, by

$$\begin{pmatrix}\varepsilon(\boldsymbol{r},220)\\ \varepsilon(\boldsymbol{r},220)\\ \cdot\\ \cdot\\ \cdot\\ \varepsilon(\boldsymbol{r},220)\end{pmatrix}=\begin{pmatrix}\sigma_{11}(\boldsymbol{s}_1) & \sigma_{22}(\boldsymbol{s}_1) & \cdots & 2\sigma_{12}(\boldsymbol{s}_1)\\ \sigma_{11}(\boldsymbol{s}_2) & \sigma_{22}(\boldsymbol{s}_2) & \cdots & 2\sigma_{12}(\boldsymbol{s}_2)\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \cdot & \cdot & \cdots & \cdot\\ \sigma_{11}(\boldsymbol{s}_9) & \sigma_{22}(\boldsymbol{s}_9) & \cdots & 2\sigma_{12}(\boldsymbol{s}_9)\end{pmatrix}\begin{pmatrix}F_{11}\\ F_{22}\\ F_{33}\\ 2F_{23}\\ 2F_{13}\\ 2F_{12}\end{pmatrix}. \qquad (5)$$

The compressive force is generated in a loading frame mounted in the Eulerian cradle of the STRESS-SPEC goniometer, Figure 4.

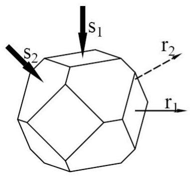

Fig. 2a

Fig. 2b

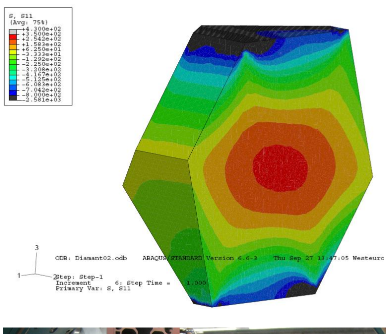

Fig. 3

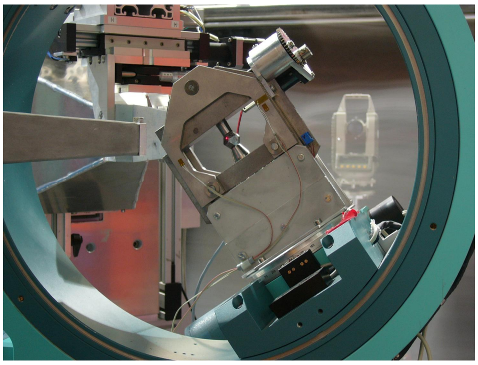

Fig. 4

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